AKTU B.Tech Sem-3

GATEWAY CLASSES

EE & EE ALLIED

EMFT

(ELECTROMAGNETIC FIELD THEORY)

Unit - 4: Lec - 3

Topics to be Covered

- Materials and Devices
- Magnetic Boundary Conditions
- PYQs

By Swati Ma'am

- M.Tech (Honours)
- B.Tech (Honours)
- GATE Qualified

MATERIALS AND DEVICES

CLASSIFICATION OF MAGNETIC MATERIALS

AKTU PYQ-2020-21

On the basis of magnetic behaviour, the materials can be classified as diamagnetic, paramagnetic and ferromagnetic.

(1) Diamagnetic Materials

The substances which when placed in a magnetic field acquire feeble magnetism opposite to the direction of the field are known as diamagnetic substances.

Examples are : Bismuth (Bi), Zinc (Zn), Copper (Cu), Silver (Ag), Gold (Au), Salt (NaCl), Water (H_2O), Mercury (Hg), Hydrogen (H_2), etc.

The permeability μ for a diamagnetic substance is less than one and susceptibility is negative.

$$\boxed{\mu < 1, \quad \chi = -ve}$$

When a diamagnetic substance is placed in a non-uniform field, then it tends to move towards the weaker part from the stronger part of the field.

(2) Paramagnetic Materials

The substances which when placed in a magnetic field, acquire **feeble** magnetism in the **direction** of magnetic field are known as paramagnetic substances.

Examples are : Aluminium (Al), Platinum (Pt), Manganese (Mn), Copper chloride ($CuCl_2$), Oxygen (O_2), solutions of salts of iron, etc.

$$\boxed{\mu > 1 \, , \quad \chi = +ve \text{ small}}$$

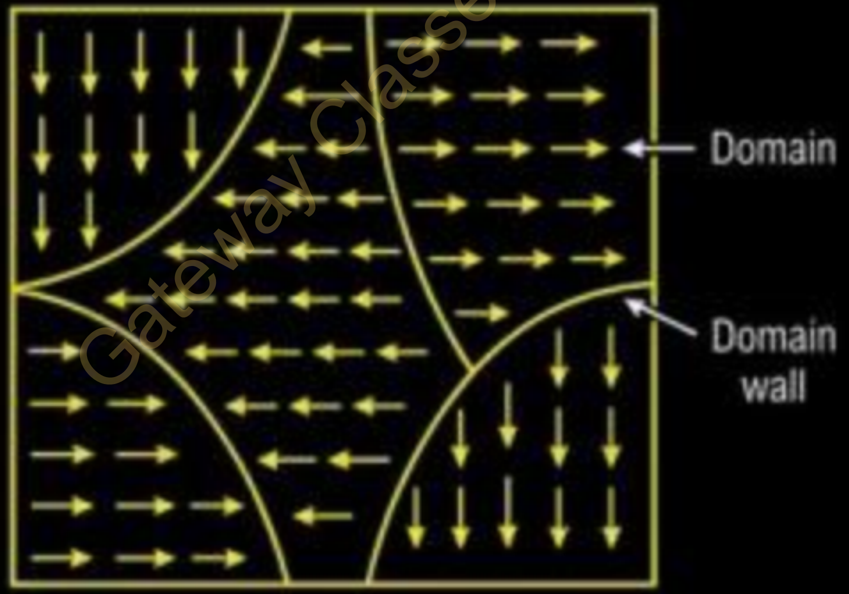
The permeability μ is greater than one and susceptibility is very small and positive.

In non-uniform magnetic field, the paramagnetic substances are attracted towards the **stronger part** of the magnetic field from the weaker part of the field.

(3) Ferromagnetic Materials

The ferromagnetic material consists of domains.

Each domain has all the magnetic moments aligned in the same direction and separated by other domains by domain wall.



The substances which when placed in external magnetic field, become **strongly magnetised** in the **direction of field** are known as ferro-magnetic substances.

The examples are : Iron (Fe), Nickel (Ni), Cobalt (Co), Magnetite (Fe_3O_4), etc.

The susceptibility of a ferromagnetic substance decreases steadily with rise in temperature according with Curie's law.

$$\boxed{\chi \propto \frac{1}{T}}$$

where T is absolute temperature.

Above certain temperature, called Curie temperature, ferromagnetic substance change to paramagnetic substances.

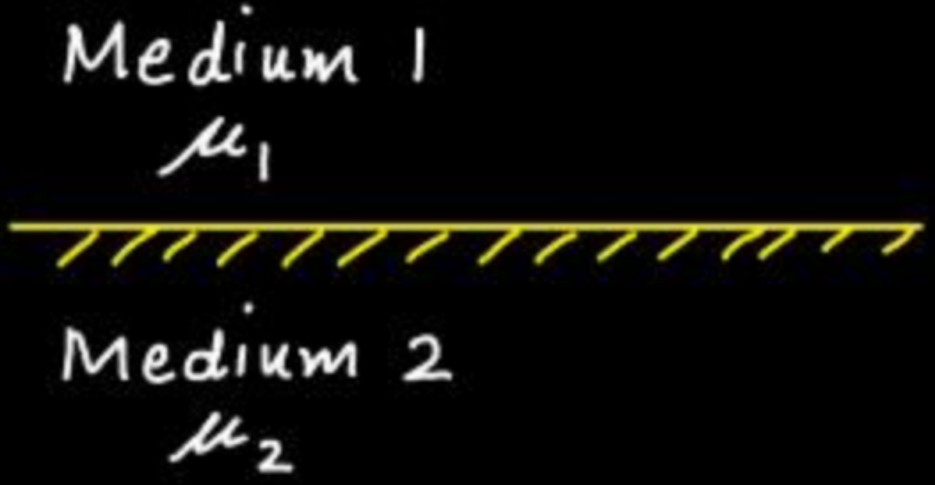
BOUNDARY CONDITIONS FOR MAGNETIC FIELD

AKTU PYQ-2023-24,2022-23,2021-22

In a single medium, the magnetic field is continuous.

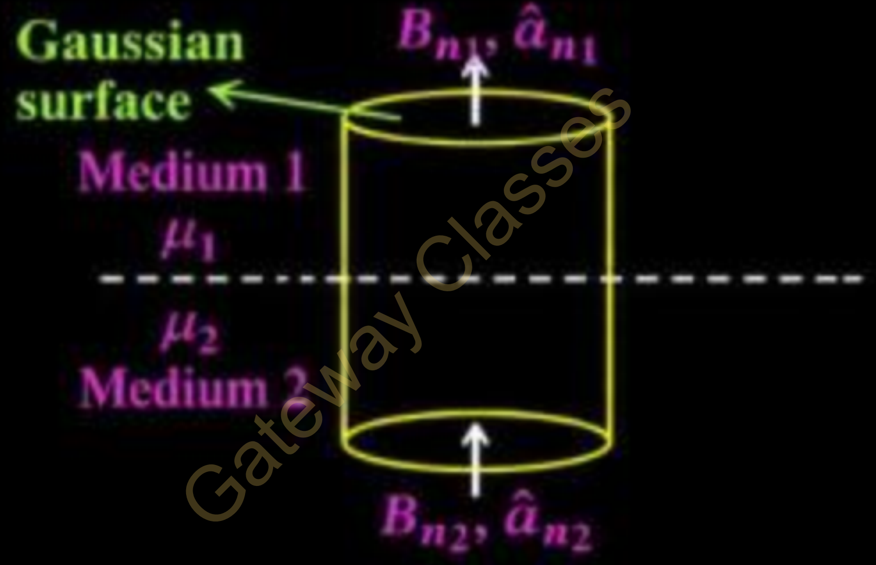
At the boundary between two magnetic materials, the magnetic field may change abruptly both in magnitude and direction.

Consider two magnetically isotropic homogeneous materials [medium 1 and medium 2] with permeabilities μ_1 and μ_2 respectively.



(1) Condition for Normal Component of B

Consider a **Cylindrical Gaussian surface** which lies half in ih the media of permeability μ_1 and μ_2. The height of the cylinder is assumed to be small so that the magnetic flux through the curved surface is negligible.



Let B_{n1} be the average component of B normal to the top surface in medium 1 and B_{n2} as the average component of B normal to the bottom surface in medium 2.

B_{n1} is an outward normal while B_{n2} is inward normal. So, B_{n1} is positive while B_{n2} is negative. Let $\hat{a}_{n1}$ and $\hat{a}_{n2}$ be the outward normals to the upper and lower surfaces respectively.

Applying Gauss's law to magnetic field, we have $\oint \vec{B} \cdot d\vec{S} = 0$

For cylindrical surface, we have $\oint \vec{B} \cdot d\vec{S} = \oint_{\text{Top}} \vec{B} \cdot d\vec{S} + \oint_{\text{Bottom}} \vec{B} \cdot d\vec{S} + \oint_{\text{Side}} \vec{B} \cdot d\vec{S} = 0$

The side of the cylinder is assumed to be very small and hence the contribution of side surface integral is taken as zero, i.e.,

$$\oint_{\text{Side}} \vec{B} \cdot d\vec{S} = 0.$$

$$\text{Therefore } \oint_{\text{Top}} \vec{B} \cdot d\vec{S} + \oint_{\text{Bottom}} \vec{B} \cdot d\vec{S} = 0$$

So, we have

$$B_{n1} \cdot \hat{a}_{n1}\ \Delta S + B_{n2} \cdot \hat{a}_{n2}\ \Delta S = 0$$

$$B_{n1}\ \overrightarrow{\Delta S} - B_{n2}\ \overrightarrow{\Delta S} = 0$$

Or

$$\boxed{B_{n1} = B_{n2}}$$

Thus, the normal components of magnetic flux densities are equal at the boundary between the two magnetic media, i.e., normal component of $\vec{B}$ is continuous across the boundary between two media.

In terms of field intensities, i.e., $\mu_1 H_{n1} = \mu_2 H_{n2}$

$$\boxed{H_{n2} = \frac{\mu_1}{\mu_2} H_{n1}}$$

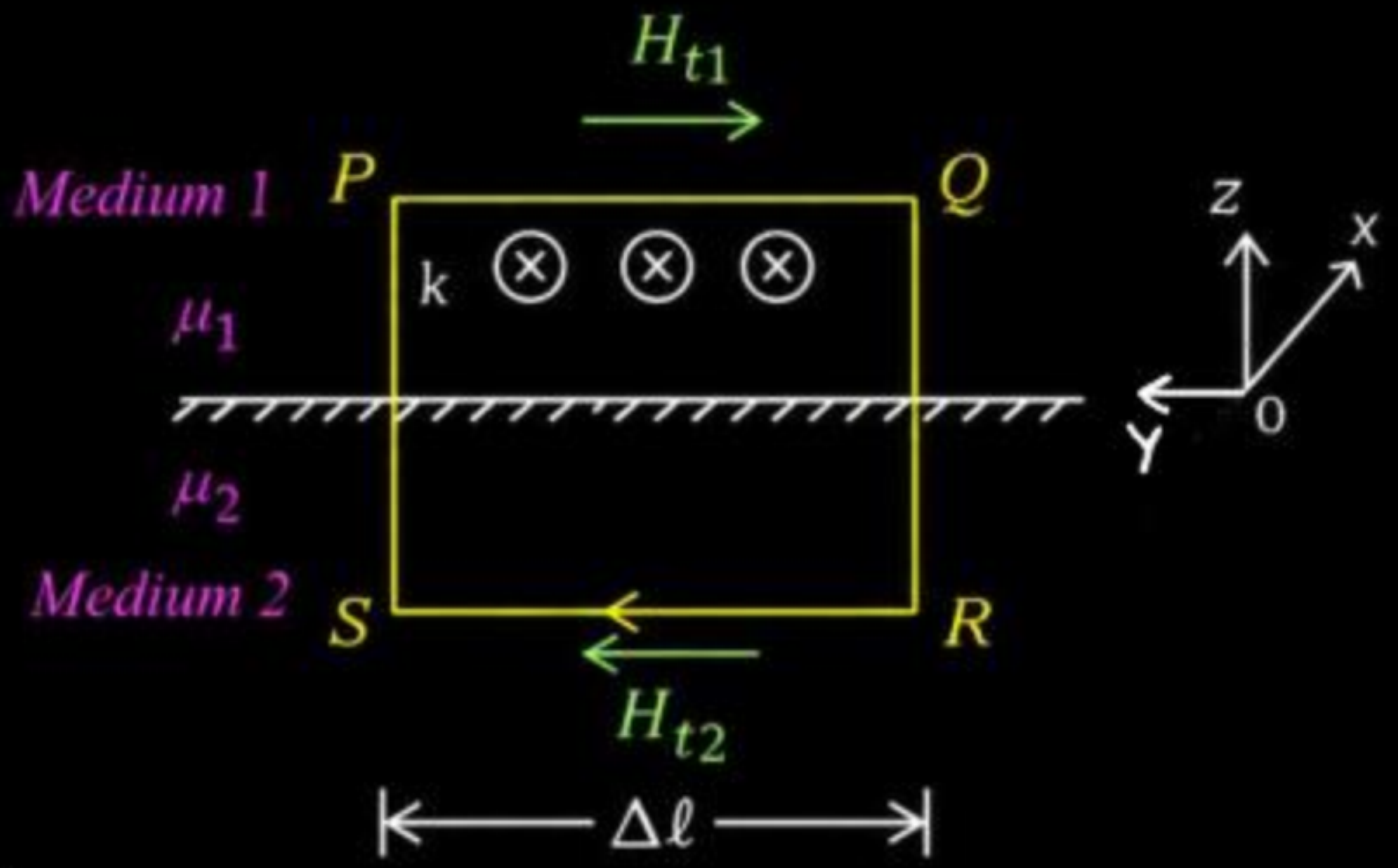
This shows that the normal component of H is discontinuous by the ratio μ_1/μ_2.

(2) Condition for Tangential Component of H

Let us consider a closed rectangular path PQRS with its long sides PQ and RS parallel to the interface and close to it.

Applying Ampere's law at the boundary

$$\oint \vec{H} \cdot \vec{dl} = I = k\Delta l$$



At boundary, there exists a sheet current of density k.

For a closed path,

$$H_{t_1}\Delta l - H_{t_2}\Delta l = k\Delta l$$

or

$$H_{t_1} - H_{t_2} = k$$

If the surface current density is zero, then

$$\boxed{H_{t1} = H_{t2}}$$

The directions of vectors are specified by the relation

$$(\vec{H}_{t_1} - \vec{H}_{t_2}) = k\hat{a}_n \quad \Rightarrow \quad \vec{H}_{t_1} - \vec{H}_{t_2} = k\hat{a}_n$$

For tangential components,

$$\vec{H}_{t1} - \vec{H}_{t2} = k \times \hat{a}_n$$

For tangential B,

$$\boxed{\frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = k}$$

Special case

If H_1 (or B_1) and H_2 (or B_2) make angles θ_1 and θ_2 with the normal to the interface between two media as shown in fig., then we have

$$H_{t1} = H_{t2}$$

or

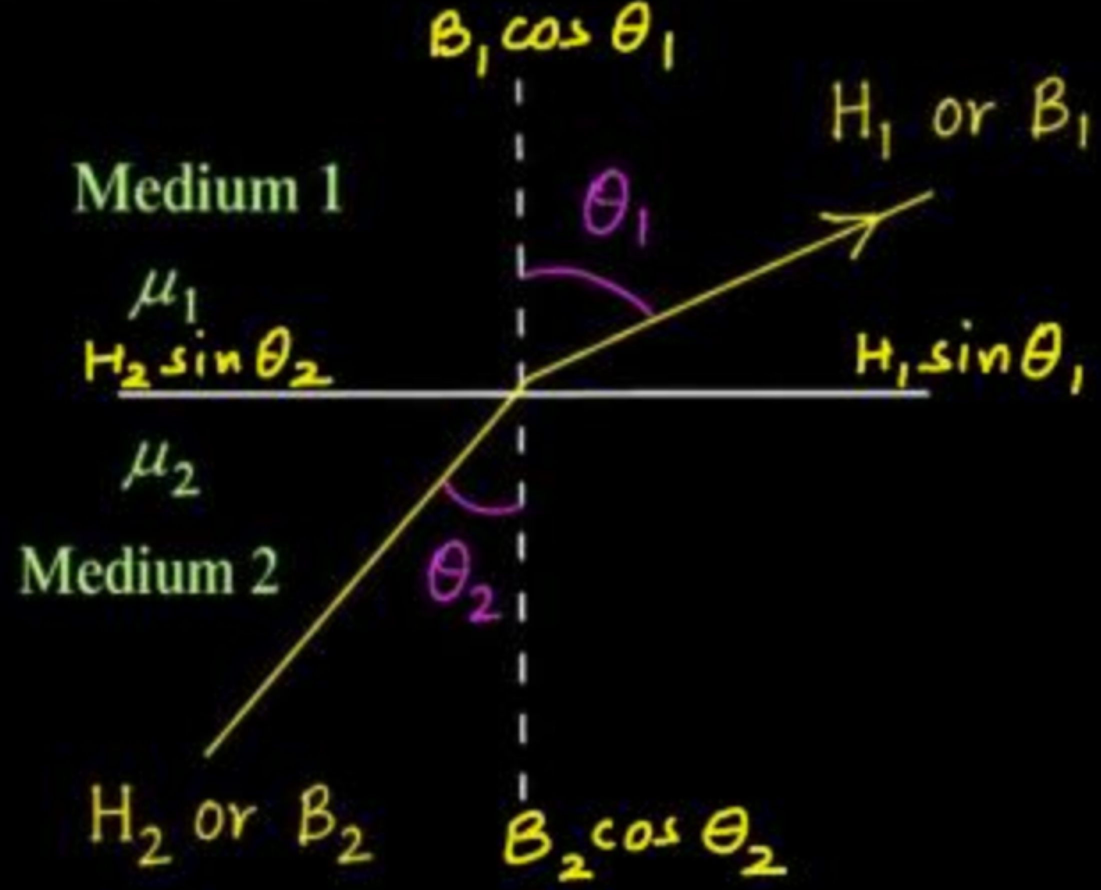
$$H_1 \sin\theta_1 = H_2 \sin\theta_2 \quad \text{— ①}$$

and

$$B_{n1} = B_{n2}$$

or

$$B_1 \cos\theta_1 = B_2 \cos\theta_2 \quad \text{— ②}$$


$B_1 \cos\theta_1$
H_1 or B_1
Medium 1
θ_1
μ_1
$H_2 \sin\theta_2$
$H_1 \sin\theta_1$
μ_2
Medium 2
θ_2
H_2 or B_2
$B_2 \cos\theta_2$

Dividing both the equations, we get ① ÷ ②

$$\frac{H_1}{B_1}\tan\theta_1 = \frac{H_2}{B_2}\tan\theta_2$$

or

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{B_1}{B_2}\frac{H_2}{H_1} = \left(\frac{B_1}{H_1}\right)\left(\frac{H_2}{B_2}\right)$$

or

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}}$$

This is the **Law of Refraction** for magnetic flux lines at the boundary with no surface current.

Gateway Classes

Empowering Learners, Transforming Futures

GATEWAY CLASSES

Thank You

GET IT ON
Google Play

Download on the
App Store

Helpline No- 7819 0058 53

AKTU B.Tech Sem-3

EE & EE ALLIED

EMFT

(ELECTROMAGNETIC FIELD THEORY)

Unit - 4: Lec - 4

Topics to be Covered

- Inductors and Inductances
- PYQs

By Swati Ma'am

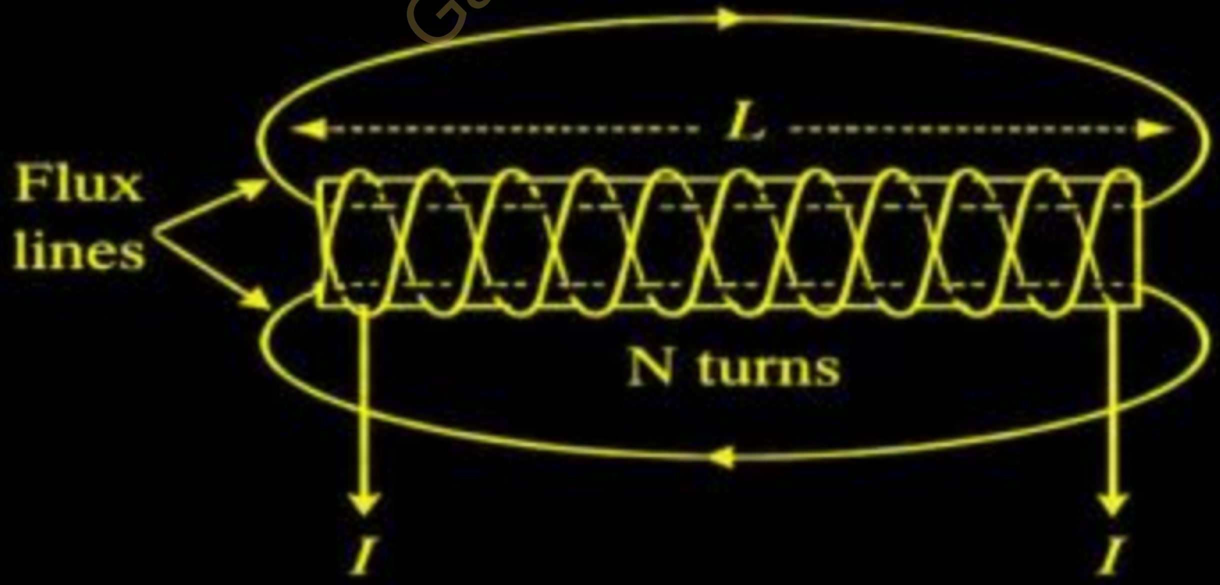
- M.Tech (Honours)
- B.Tech (Honours)
- GATE Qualified

INDUCTORS AND INDUCTANCES

AKTU PYQ-2023-24, 2021-22

A capacitor stores energy in an electric field. Similarly, inductor is a device which stores energy in magnetic field.

Solenoids, coils, loops, etc. are examples of inductors.
Consider a solenoid coil in which a current I is passing.



Now, magnetic lines are produced by the current. Each line passes through the entire solenoid.

Let N be the total number of turns in the solenoid. The total flux linked with the solenoid would be $N\Phi_m$ where Φ_m is the flux linked with a coil. **The inductance is defined as the ratio of total magnetic flux linked to the current producing the flux.**

Thus, Inductance,

$$\boxed{L = \frac{N\phi_m}{I}}$$

L = self inductance

N = total number of turns, Φm = flux linked with a coil

I = current passing through each turn.

The unit of inductance is henry.

1 henry = 1 wb/A

INDUCTANCE OF A SOLENOID

Consider a long solenoid of length l metre and uniform cross-sectional area A metre^2. Let N be the number of turns. Suppose a current I amp. flows through it. The magnetic field inside the solenoid is given by

$$\boxed{B = \frac{\mu N I}{l}}$$

where μ is the permeability of the material (usually air).

For air

$\mu = \mu_0 = 4\pi \times 10^{-7}$ (H/m)

The total flux linked with the solenoid is given by

Total flux = $N\,\Phi_m = N\int_S \mathbf{B}\cdot d\mathbf{S} = N\,B\,A$

Substituting the value of B

Total flux = $$N\left(\frac{\mu N I}{\ell}\right)A = \frac{\mu N^2 A I}{\ell}$$

Now, Inductance, $$L = \frac{Total\ flux}{current} = \frac{\mu N^2 A I}{I\,\ell}$$

or

$$\boxed{L = \frac{\mu N^2 A}{\ell}}$$

If n be the number of turns per unit length, then total number in the solenoid would be

$$N = nl$$

The inductance in this case is given by

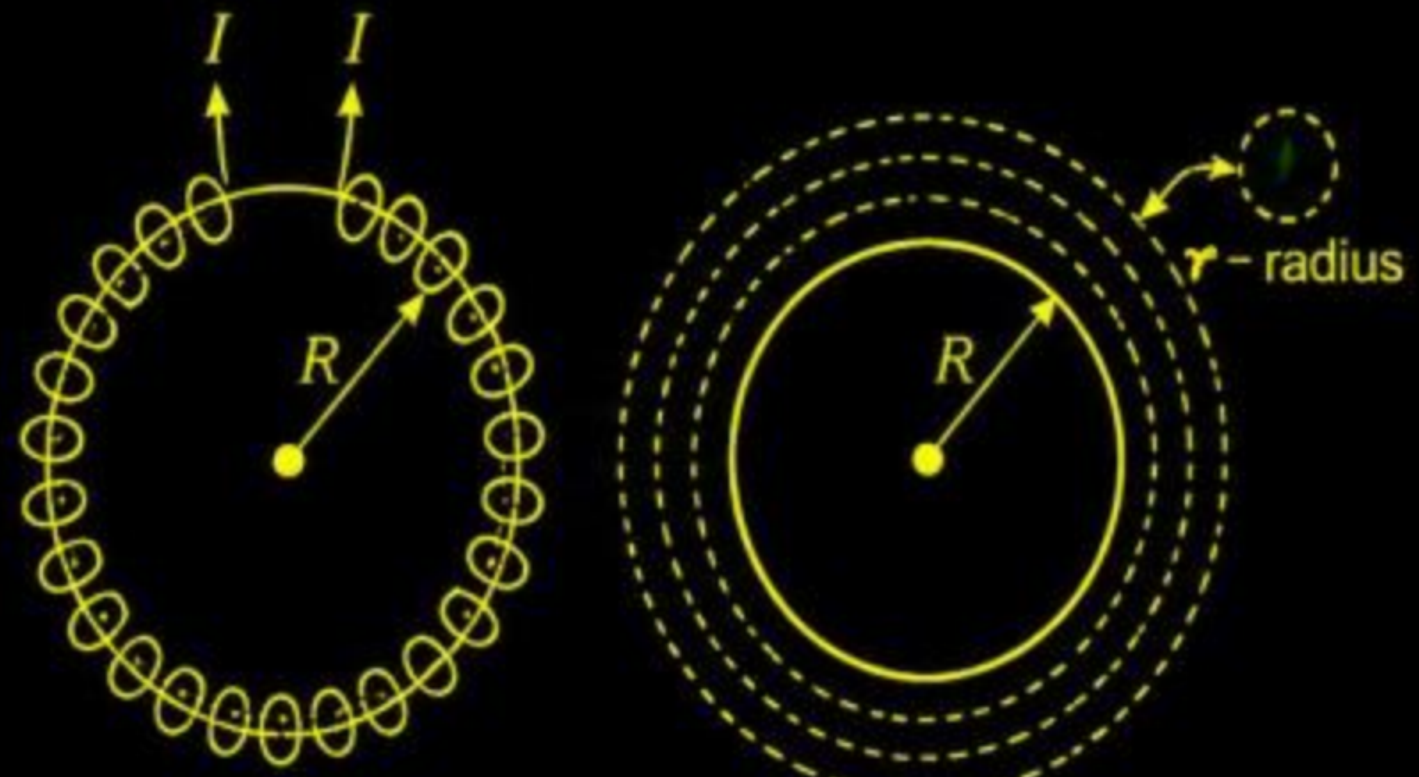
$$L = \frac{\mu\,(nl)^2 A}{l}$$

or

$$\boxed{L = \mu n^2 l\,A}$$

INDUCTANCE OF A TOROID

A toroid may be regarded as a solenoid bent into a circular form so that its ends are joined together. Thus, it may be called as **endless solenoid.**



(a) (b)

Fig shows a toroid having circular cross-section.

Let R be the mean radius of the toroid and N, the number of turns in it. The magnetic flux lines form concentric circles.

Let I amp. be the current flowing in the toroid and B, the magnetic field produced due to this current on the circular path. Then, according to Ampere's law $\oint \vec{H}\cdot\vec{dl} = I_{enclosed}$ or $I \Rightarrow \oint \frac{\vec{B}}{\mu}\cdot\vec{dl} = I \quad (\because B = \mu H)$

$$\oint \vec{B}\cdot\vec{dl} = \mu \times \text{(current enclosed by the path)}$$

$$= \mu N I$$

As the magnetic field B is tangential around the circular path of radius R, we have

$B \times 2\pi R = \mu N I$

$$B = \frac{\mu N I}{2\pi R}$$

Total Flux = B N A

$$N\phi_m = \left(\frac{\mu N I}{2\pi R}\right) N A$$

$$N\phi_m = \frac{\mu N^2 I A}{2\pi R}$$

$$L = \frac{N\Phi_m}{I} = \frac{\mu N^2 A}{2\pi R}$$

$$L = \frac{\mu N^2 A}{2\pi R} \quad \text{henry}$$

$$L = \frac{\mu N^2 \pi r^2}{2\pi R}$$

$$\boxed{L = \frac{\mu N^2 r^2}{2R}}$$

r : radius of cross - section of toroid

INDUCTANCE OF COAXIAL CABLE

AKTU PYQ- 2021-22

Consider a distance r between the conductors. The magnetic induction B between the conductors at a distance r is given by

$$\Phi_m = \int_S \vec{B} \cdot \vec{dS} \quad \text{where } \vec{dS} = dr\,dz\,\hat{a}_\phi$$

$$\Phi_m = \int_0^l \int_a^b \left(\frac{\mu_0 I\,\hat{a}_\phi}{2\pi r}\right) \cdot (dr\,dz\,\hat{a}_\phi) = \int_0^l \int_a^b \frac{\mu_0 I}{2\pi r}\,dr\,dz$$

$$= \frac{\mu_0 I}{2\pi} \int_0^l dz \int_a^b \frac{dr}{r}$$

$$= \frac{\mu_0 I}{2\pi} \times l \times \log_e\left(\frac{b}{a}\right)$$

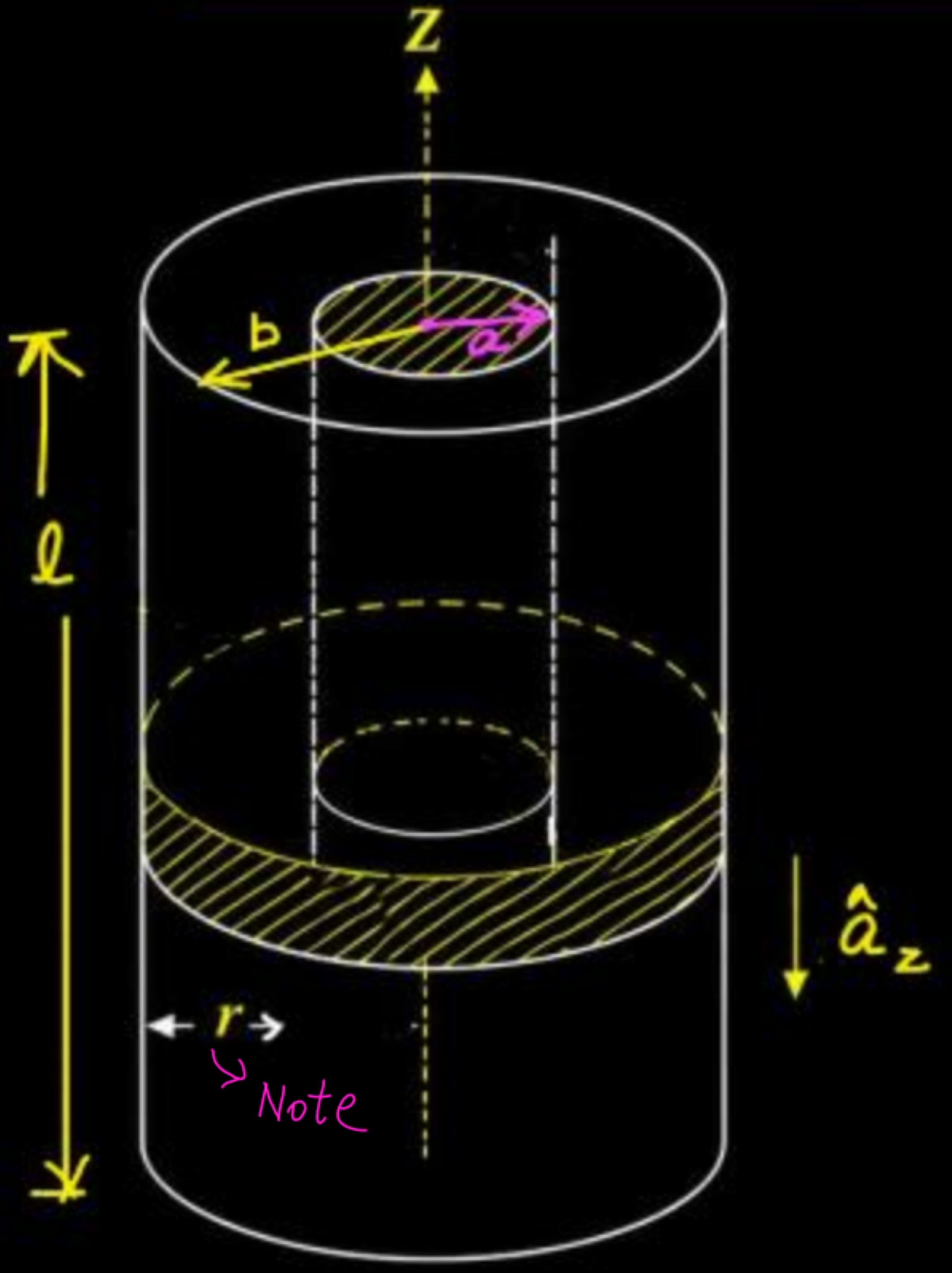
Now inductance,

$$L = \frac{\Phi_m}{I} = \frac{\mu_0 l}{2\pi} \log_e\left(\frac{b}{a}\right)$$

Inductance per unit length of the cable

$$L_1 = \frac{L}{l}$$

$$\boxed{L_1 = \frac{\mu_0}{2\pi} \log_e\left(\frac{b}{a}\right)} \quad H|m$$



Gateway Classes

GATEWAY CLASSES

Empowering Learners, Transforming Futures

Thank You

GET IT ON
Google Play

Download on the
App Store

Helpline No- 7819 0058 53

AKTU B.Tech Sem-3

EE & EE ALLIED

EMFT

(ELECTROMAGNETIC FIELD THEORY)

Unit - 4: Lec - 5

Topics to be Covered

- Magnetic Energy
- PYQs

By Swati Ma'am

- M.Tech (Honours)
- B.Tech (Honours)
- GATE Qualified

GATEWAY CLASSES

MAGNETIC ENERGY

ENERGY DENSITY IN MAGNETIC FIELD

AKTU PYQ- 2023-24,2022-23

Consider a very long solenoid of length l and cross-sectional area A.

When a current flows in it, a magnetic field is established. This field is uniform inside and negligible outside. So, the volume associated with the magnetic field is Al.

We have seen that the amount of workdone in establishing a current I_0 in the solenoid is

$$\left(\frac{1}{2}\right) L\, I_0^{\,2}$$

where L is the inductance of solenoid.

The workdone is stored as energy in the magnetic field.

$$\boxed{U = \text{energy stored} = \frac{1}{2} L\, I_0^{\,2}}$$

The inductance of the solenoid is given by,

$$L = \mu_0 n^2 A\ell$$

where n is number of turns in solenoid is given by

$$U = \frac{1}{2}\left(\mu_0 n^2 A\ell\right) I_0^2 = \frac{1}{2}\,\frac{\left(\mu_0 n I_0\right)^2}{\mu_0} A\ell$$

$$U = \frac{1}{2}\,\frac{B^2}{\mu_0}\,A\ell \qquad (:\; B = \mu_0 n I_0)$$

Energy

Energy density (u) $= \frac{U}{V} = \frac{U}{Al}$

$$u = \frac{U}{Al}$$

$$= \frac{1}{2}\frac{B^2 Al}{\mu_0 Al}$$

$$\boxed{u = \frac{B^2}{2\mu_0}} \quad joule/m^3$$

Q.6 A 100-turn closely wound circular coil of radius 10 cm carries a current of 3.2 A. The coil has a moment of inertia of 0.1 kg(m)^2. The coil is placed in a vertical plane and is free to rotate about a horizontal axis which coincides with its diameter. A uniform magnetic field of 2 T in the horizontal direction exists such that initially the axis of the coil is in the direction of the field. The coil rotates through an angle of 90 degrees under the influence of the magnetic field. What is the angular speed acquired by the coil when it has rotated by 90 degrees?

AKTU PYQ- 2024-25

$$\boxed{\Delta U = \frac{1}{2} J\omega^2 = m B}$$

change in energy converted into rotational kinetic energy

where $m = N I A$

$A = \pi r^2$

m - Magnetic moment of the coil
J - Moment of inertia
B - Magnetic field
ω - Angular speed
N - No. of turns
I - Current
r - radius of coil

Now,

$$\begin{aligned} m &= N\,I\,A \\ &= N\,I\,(\pi\, r^2) \end{aligned}$$

$$\therefore \quad m = 100\,(3.2)\,(3.14)\,(10 \times 10^{-2})^2$$

$$= 10.05$$

$$\frac{1}{2}\,J\,\omega^2 = m\,B$$

$$\frac{1}{2}\,(0.1)\,\omega^2 = 10.05 \times 2$$

$$\omega^2 = \frac{10.05 \times 2 \times 2}{0.1}$$

$$\omega^2 = 402$$

$$\boxed{\omega = 20.05 \ \text{rad/s}}$$

Ans

Gateway Classes

Empowering Learners, Transforming Futures

GATEWAY CLASSES

Thank You

GET IT ON
Google Play

Download on the
App Store

Helpline No- 7819 0058 53